Toward Fabricating Surfaces as Patchworks of Pantographic Lattices

James Mallos, Sculptor, Washington DC, USA

Abstract

An elegant way to fabricate a patch of curved surface is to constrain the perimeter of a patch of wire mesh. The technique is familiar to sculptors; it is known by the term *gridshell* among architects, and *pantographic lattice* among metamaterial technologists. This paper presents results of the author's preliminary research on fabricating 3D-printed surfaces out of rudimentary pantographic lattice patches that can be quickly printed flat, and then seamed together (constraining the perimeters of the patches at the same time) by a simple chain stitching technique.

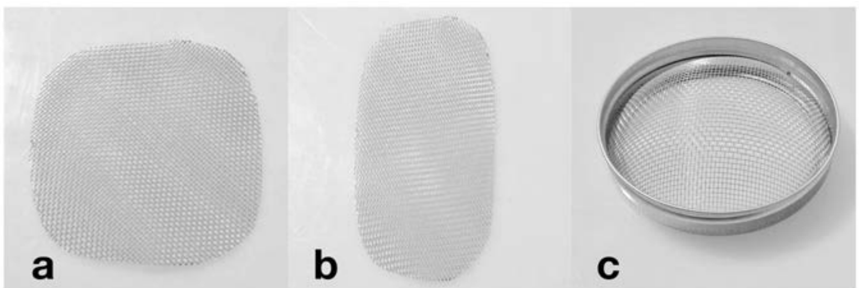

Figure 1: *Shape changer: a) a patch of wire mesh; b) the same patch conformed to a new perimeter; c) the same patch conformed to a perimeter incompatible with planarity.*

Introduction

It is a common observation that woven fabric drapes differently from, say, plastic film, because it offers little resistance to shear—and the draping is sensitive to the orientation of the grain of the fabric. When a stiff fabric is used, say, the wire mesh used in a kitchen strainer [1], the effect is so pronounced that merely constraining the perimeter of a patch of fabric to a shape incompatible with planarity causes it belly out into a three-dimensionally curved surface (Figure 1.) In recent decades this shape-changing behavior has caught the interest of both architects at the large scale (gridshells) and metamaterial technologists at the small scale (pantographic lattices). Computer scientists interested in modeling the dynamics of hair and fur have at the same time contributed fast algorithms that aid the computational modeling of grids. This paper presents some preliminary results in the author's attempt to harness this now well-understood phenomenon to make large models quickly in small 3D printers—a commonly felt need in educational and shared-use environments.

Background

In 1878 Tchebyshev [16, 13, 6] described how a woven fabric—which he modeled as a pin-jointed, equilateral rhombic grid ('fishnet' in fabric parlance)—can be made to conform to a portion of a curved surface. Starting from two crossing curves on the surface, the discrete version of his geometrical construction can be accomplished with just a compass. Tchebyshev also derived a fundamental limit on the amount of curvature such a construction can span: for example, it is not possible to wrap a sphere with fishnet without producing a singularity. His purely geometric model still gives useful first approximations for gridshells and pantographic lattices.

In 1970, the erection of Frei Otto's Mannheim Gridshell [8] sparked the architectural interest in gridshells that has continued to this day. Many gridshells utilize the shape change effect, being assembled flat and erected by compressing the perimeter, usually with lifting assistance from cranes above or an inflated membrane below. As a practical matter, architectural gridshells are usually limited to structural members that are straight when on the ground; and the erected gridshell needs to be rigidified against asymmetric loads and rendered water-tight in some way. (In 3D printing we are not limited to members that are straight when printed; the present research does not address how to to rigidify the assembled model or make it water-tight.)

Mathematical modeling of gridshells is now quite advanced. Bending and torsion of structural members that are not round or square in cross-section [3], or are curved when on the ground [11], can now be modeled. Algorithms originally developed in computer graphics to realistically model hair in motion have made gridshell modeling faster [2, 1]. Algorithms have also been developed that can cover a curved surface with a patchwork of grids [9] (thereby escaping Tchebyshev's curvature limit,) design wire meshes with interactive user input and a degree of freedom in approximation to the guiding form [5], automatically place singularities (i.e., vertices that are not 4-valent) to extend a Tchebyshev net to cover an entire surface [14], and design freeform gridshells using the theoretical minimum of material [12].

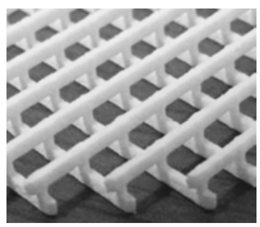

Figure 2: *A pantographic lattice 3D-printed for metamaterial research [4].*

More recently, metamaterial engineers have been interested in pantographic lattices [4] (miniature gridshells in the flattened state) that can be realized by 3D printing (Figure 2). There has also been metamaterial research into the shape change effect in pantographic lattices with curved members [7].

The Goal

FDM 3D printing is slow. Build volume is expensive. These limitations are acutely felt where the user has limited machine time, as in a classroom or maker space. It would be nice to be able to quickly make models larger than the build volume in these situations—even if some post-printing assembly were required. The shape changing effect in pantographic lattices seems to offer some hope to get around this problem since flat objects can be quickly printed on the build plate. Metamaterial research [4] has shown that pantographic lattices can be successfully 3D printed (Figure 2.) Taking a hint from 'zippable' shape representations [15] it would seem possible to join 3D lattice patches by chain stitching loops printed along their perimeter. Chain stitching is a very easy hand technique that has traditionally been used to 'zip' and 'unzip' saddle bags [10] (Figure 3.)

Main Challenges

Fast-to-Print Pantographic Lattices

The pantographic lattice of Figure 2 is too refined to print quickly. A faster alternative would be to extrude the upper layer of rods at a speed and temperature that produces small cylinder-to-cylinder welds to the lower

Figure 3: *A traditional chain stitch closure on a saddlebag [10].*

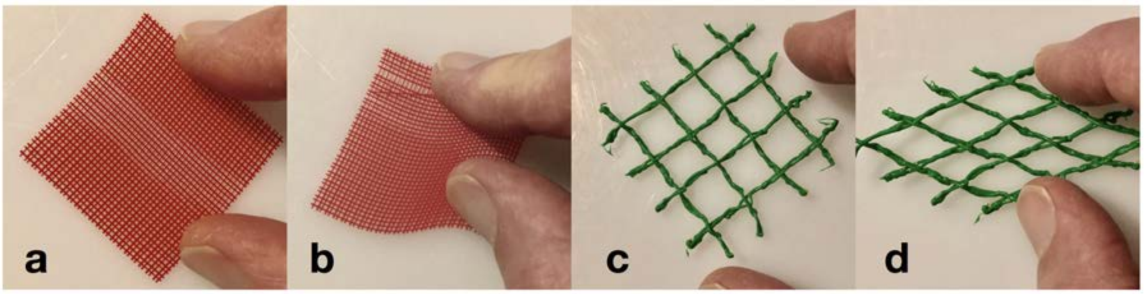

Figure 4: *Fast Lattices: a-b) two-pass; c-d) four-pass.*

rods (Figure 4a). Unfortunately, my trials found no happy medium where scissor action was loose enough to shear without curving, yet welds were strong enough to bind the fabric securely together (Figure 4b.)

A four-pass lattice seems more promising, though I have only sketched it with a 3D pen (Figure 4c-d.) In a four-pass lattice the extruder retraces each path, making z-hops (vertical jumps) over rods it is crossing (thereby avoiding a weld,) and presses down along any rod it is retracing (thereby producing a linear weld.) The result is like a net of twined braiding, but with linear welds rather than twists joining the paired strands. The welds are not severely loaded by the scissoring action, so they do not need to be especially secure.

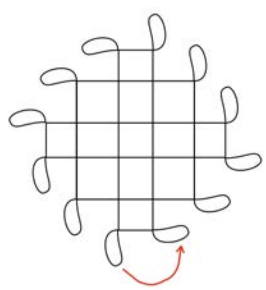

Figure 5: *Using a chain stitch to constrain the perimeter of a 3D-printed pantographic lattice: each terminal loop encloses, in this case, its counterclockwise neighbor. The last stitch must be locked in another way.*

Chain Stitching the Seams

I envision constraining the perimeter of the patches, and seaming them together, by chain stitching loops that are printed integral to the lattices (Figure 5.) The loops are sized to properly constrain the lattice only after the loops of the adjoining patch are intertwined in the manner of a chain stitched seam in crochet. Thus each seam is in effect chain-stitched twice. This is actually an advantage. It is always possible to find a spanning tree in any graph on the sphere (by breaking certain edges at their midpoint), and thus to tour all its edges (seams) in a closed loop that goes along each seam twice. Given such a plan of seaming, only the last stitch needs to be locked.

Summary and Conclusions

I have sketched a novel way to fabricate larger shapes more rapidly using the 3D printing resources available to schools and maker spaces. Much needs to be done to bring this to fruition.

References

[1] Changyeob Baek, Andrew O Sageman-Furnas, Mohammad K Jawed, and Pedro M Reis. Form finding in elastic gridshells. *Proceedings of the National Academy of Sciences*, 115(1):75–80, 2018.

[2] Miklós Bergou, Max Wardetzky, Stephen Robinson, Basile Audoly, and Eitan Grinspun. Discrete elastic rods. In *ACM SIGGRAPH 2008 papers*, pages 1–12. 2008.

[3] Bernardino D'Amico. *Timber Grid-shell Structures: form-finding, analysis and optimisation*. PhD thesis, Edinburgh Napier University, 2015.

[4] Francesco dell'Isola, Ivan Giorgio, Marek Pawlikowski, and Nicola Luigi Rizzi. Large deformations of planar extensible beams and pantographic lattices: heuristic homogenization, experimental and numerical examples of equilibrium. *Proceedings of the Royal Society A: Mathematical, Physical and Engineering Sciences*, 472(2185):20150790, 2016.

[5] Akash Garg, Andrew O Sageman-Furnas, Bailin Deng, Yonghao Yue, Eitan Grinspun, Mark Pauly, and Max Wardetzky. Wire mesh design. *ACM Transactions on Graphics*, 33(4), 2014.

[6] Étienne Ghys. Sur la coupe des vêtements. Variation autour d'un thème de Tchebychev. *L'Enseignement Mathématique*, 57(1):165–208, 2011.

[7] Ivan Giorgio, Alessandro Della Corte, Francesco Dell'Isola, and David J Steigmann. Buckling modes in pantographic lattices. *Comptes rendus Mecanique*, 344(7):487–501, 2016.

[8] Ian Liddell. Frei Otto and the development of gridshells. *Case Studies in Structural Engineering*, 4:39–49, 2015.

[9] Yannick Masson. *Existence and construction of Chebyshev nets and application to gridshells*. PhD thesis, 2017.

[10] Edith Eliza Ames Norton. *Rugs in their native land*, page 135. Dodd, Mead, 1910.

[11] Julian Panetta, MINA Konaković-Luković, Florin Isvoranu, Etienne Bouleau, and Mark Pauly. X-shells: A new class of deployable beam structures. *ACM Transactions on Graphics (TOG)*, 38(4):1–15, 2019.

[12] Davide Pellis and Helmut Pottmann. Geometry and statics of optimal freeform gridshells. 2020.

[13] Eugene Vladimirovich Popov. Geometric approach to Chebyshev net generation along an arbitrary surface represented by nurbs. In *International Conference Graphicon*, 2002.

[14] Andrew O Sageman-Furnas, Albert Chern, Mirela Ben-Chen, and Amir Vaxman. Chebyshev nets from commuting polyvector fields. *ACM Transactions on Graphics (TOG)*, 38(6):1–16, 2019.

[15] Christian Schüller, Roi Poranne, and Olga Sorkine-Hornung. Shape representation by zippables. *ACM Transactions on Graphics (TOG)*, 37(4):1–13, 2018.

[16] Pafnuty L Tchebychev. Sur la coupe des vêtements. In *Association française pour l'avancement des sciences, Congres de Paris*, pages 154–155, 1878.